

WHY ORDER DOES NOT MATTER: AN APPEAL TO IGNORANCE

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Within the limited field of research on teachers' probabilistic knowledge, incorrect, inconsistent and even inexplicable responses to probabilistic tasks are most often accounted for by utilizing theories, frameworks and models, which are based upon heuristic and informal reasoning. More recently, the emergence of new research based upon informal logical fallacies has been proving effective in accounting for certain normatively incorrect responses to probabilistic tasks. This article contributes to this emerging area of research by demonstrating how a particular informal logical fallacy, known as "an appeal to ignorance," can be used to account for a specific set of normatively incorrect responses to a novel probabilistic task, which were provided by prospective elementary and secondary mathematics teachers.

Keywords: Cognition; Probability; Teacher Education—Preservice; Teacher Knowledge

The objective of this article, in general, is to contribute to the limited amount of research on (prospective) "teachers' probabilistic knowledge" (Jones, Langrall, & Mooney, 2007, p. 933; Stohl, 2005). In specific, the objective of this article is to contribute to the well-established domain of research, which accounts for (through various theories, models and frameworks) incorrect, inconsistent, sometimes inexplicable responses to a range of probabilistic tasks (Abrahamson, 2009; Chernoff, 2009; Kahneman & Tversky, 1972; Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; LeCoutre, 1992; Tversky & Kahneman, 1974).

To meet the general and specific objectives stated, prospective teachers were asked to determine and justify which of two student responses provided the correct answer and explanation to a question, which involved determining the probability that a three-child family has two daughters and one son. In addition to contributing a twist to a task recently introduced to the research literature, we also utilize a novel lens to account for certain responses to that task. In our analysis, we demonstrate that logically fallacious reasoning, more specifically, in this instance, an appeal to ignorance (i.e., there is no evidence for p ; therefore, *not-p*) accounts for certain prospective teachers' normatively incorrect responses to the task. By meeting the general and specific objectives presented, this article will correspond and contribute to a continuation or, stated in terms associated with the theme of PMENA 2012, a *transition*—from heuristic reasoning (e.g., Kahneman & Tversky, 1972) and informal reasoning (e.g., Konold, 1989) to logically fallacious reasoning (e.g., Chernoff & Russell, 2011a, 2011b, in press)—in how researchers account for particular prospective teachers' responses to probabilistic tasks.

A Brief Summary of Prior Research

Research into probabilistic thinking and the teaching and learning of probability has, in the past, seen a focus on normatively incorrect responses. Worthy of note, the focus on normatively incorrect responses does not, in any way, suggest a negative view of the mind (Kahneman, 2011). The theories, models and frameworks associated with heuristic and informal reasoning—rooted in the notions of conceptual analysis (Thompson, 2008, Von Glasersfeld, 1995), grounded theory (Strauss & Corbin, 1998) and abduction (Peirce, 1931)—have, traditionally, accounted for normatively incorrect responses to probabilistic tasks within the field of mathematics education. Chernoff (2009a, 2009b, 2009c, 2011, in press) and Chernoff and Russell (2011a, 2011b, in press) have provided detailed accounts of the theories, models and frameworks associated with heuristic and informal reasoning in the field of mathematics education and are recommended—given the 8-page limitation associated with the present venue—to the reader.

More recently, a burgeoning area of research suggests that fallacious reasoning, more specifically, the use of informal logical fallacies, can account for certain normatively incorrect responses to probabilistic tasks. For example, Chernoff and Russell (2011a) demonstrated that certain prospective mathematics

teachers—when asked to identify which event (i.e., outcome or subset of the sample space) from five flips of a fair coin was least likely to occur—did not use the representativeness heuristic (Kahneman & Tversky, 1972), the outcome approach (Konold, 1989) or the equiprobability bias (Lecoutre, 1992), but, instead, utilized a particular informal logical fallacy, the fallacy of composition: when an individual infers something to be true about the *whole* based upon truths associated with *parts* of the *whole* (e.g., coins (the parts) are equiprobable; events (the whole) are comprised of coins; therefore, events are equiprobable, which is not necessarily true). Worthy of note, the fallacy of composition accounted for both normatively correct and incorrect responses to the new relative likelihood comparison task.

In subsequent research, Chernoff and Russell (2011b, in press) applied the fallacy of composition to a more traditional relative likelihood comparison. Prospective mathematics teachers were asked to determine which of five possible coin flip sequences—not events—were least likely to occur. As was the case in their prior research (e.g., Chernoff & Russell, 2011a), the fallacy of composition accounted for normatively incorrect responses to the task. More specifically, the researchers demonstrated that participants reference the equiprobability of the coin, note that the sequence is comprised of flips of a fair coin and, as such, fallaciously determine that the sequence of coin flips should also have a heads to tails ratio of one to one. In other words, the properties associated with the fair coin (the parts), which make up the sequence (the whole), are expected in the sequence. Once again, the fallacy of composition, not the traditional theories, models and frameworks associated with heuristic and informal reasoning, accounted for certain normatively incorrect responses to a probabilistic task.

Chernoff and Russell (2011a, 2011b, in press) contend, based on their research utilizing the fallacy of composition, that they have (re)opened a new area of investigation for those researching probabilistic thinking and the teaching and learning of probability. However, they also contend that more research will allow individuals to determine to what extent informal logical fallacies and fallacious reasoning can account for normatively incorrect responses to a variety of probabilistic tasks. The former and latter contentions have provided the motivation for us to determine whether or not another informal logical fallacy, an appeal to ignorance, can account for normatively incorrect responses to a probabilistic task that has recently been introduced to the research literature.

Theoretical Framework

Of the numerous informal fallacies that could, potentially, be utilized as a theoretical framework (e.g., equivocation, begging the question, the fallacy of composition, the fallacy of division and others), our analysis of results will rely, specifically, on one particular informal logical fallacy: *an appeal to ignorance*, which, essentially, “is an argument for or against a proposition [p] on the basis of a lack of evidence against or for it” (Curtis, 2011, para. 3). However, and worthy of note, an appeal to ignorance can come in one of two forms: (1) there is no evidence against p . therefore, p and (2) there is no evidence for p . therefore, *not-p*. To be more specific, our analysis of results will rely on the second form of an appeal to ignorance: there is no evidence for p . therefore *not-p*. Stated in more colloquial terms, the reader may be familiar with the following phrase: “the absence of evidence is not evidence of absence.” For example, consider the following question: Is there a lawn mower in my garage? If one does not look inside my garage, the absence of evidence does not amount to evidence of an absence of a lawn mower because there may, in fact, be a lawn mover in my garage. Thus, and stated once again in colloquial terms, an individual who declares the absence of evidence as evidence of absence is employing (the second form of) an appeal to ignorance. Our attention, in the analysis of results, will focus on a set of individuals utilizing the absence of evidence as evidence of absence. Stated, in the terms of our example, our analysis of the results will focus on those individuals who do not look inside my garage and use their lack of evidence of a lawn mower in my garage to declare that there is no lawn mower in my garage. However, our research does not focus on garages and lawn mowers; instead, our research focuses on whether order matters or order does not matter for a particular probabilistic task.

The Jane or Dianne Task

The Jane or Dianne task, presented below in Figure 1, represents an alteration to the original “two boys and a girl task” (Chernoff & Zazkis, 2011, p. 21), which was utilized in previous research (Chernoff & Zazkis, 2011) and introduced by Chernoff and Zazkis (2010).

What is the probability that a three-child family has two daughters and one son?

Jane’s explanation: Out of the four possible outcomes (3 daughters, 0 sons; 2 daughters, 1 son; 1 daughter, 2 sons; and 0 daughters, 3 sons) only one outcome (2 daughters, 1 son) is favourable, so the probability is one-fourth.

Dianne’s explanation: Out of the eight possible outcomes (daughter, daughter, daughter; daughter, daughter, son; daughter, son, daughter; son, daughter, daughter; daughter, son, son; son, daughter, son; son, son, daughter; son, son, son) only three outcomes (daughter, daughter, son; daughter, son, daughter; son, daughter, daughter) are favourable, so the probability is three-eighths

_____’s explanation is correct because.....

Figure 1: The Jane or Dianne task

Fundamentally, the two boys and a girl task is the same as the Jane or Dianne task. In other words, the core of the task, the probability question, that is, what is the probability that a three-child family has two daughters and one son, is the same in both tasks. Previously, the task was utilized in order to elicit insight into prospective secondary school mathematics teachers’ pedagogical approaches. In this new version of the task, however, the focus is not pedagogical, but, rather, on which response prospective mathematics teachers deem mathematically correct and, relatedly, which explanation is deemed appropriate. Stated in more general terms, the task has been altered in order to contribute to the limited amount of research on what Jones, Langrall, and Mooney (2007) called “teachers’ probabilistic knowledge” (p. 933).

Participants

The ($n =$) 130 participants in our research were comprised of 52 (40%) prospective elementary school teachers (PESTs) and 78 (60%) prospective secondary school teachers (PSSTs). The PESTs were enrolled in a methodology course designed for teaching elementary school mathematics and the PSSTs were enrolled in a methodology course designed for teaching secondary school mathematics. The 52 PESTs were from two different classes (each containing approximately 25 students) and, similarly, the 78 PSSTs were from three different classes (each containing approximately 25 students). For both the PESTs and the PSSTs, the topic of probability had not yet been addressed in their methodology courses. Instead, content, strategies and approaches garnered from research and practice related to the teaching and learning of probability were addressed after the data for this research was collected. To collect the data, participants were asked and given as much time as required to determine, via written response, which of the two explanations, Jane’s or Dianne’s, was correct and, further, to justify their choice also via written response.

Results

As seen in Table 1 below, there was, roughly, an even split between those individuals who declared and explained why Jane and Dianne’s response was correct. Roughly half (51%) of the participants declared that Dianne’s explanation was correct. More specifically, 40 of the 78 PSSTs (51%) and 26 of the 52 (50%) of the PESTs chose Dianne and her explanation. Thus, there was little difference between the percentage of PESTs and PSSTs that chose Dianne and her explanation. Worthy of note, not all 66 of the 130 participants provided an appropriate justification for why Dianne’s explanation was correct. Interestingly, 9 of the participants (7%) chose an option that was not presented to them. These individuals

may have indicated that neither choice and explanation was correct or that both of the choices and explanations were correct. These individuals have been placed in the ‘Other’ column in Table 1. As mentioned, there are certain pagination limitations associated with the current venue and, as such, the responses from those individuals who chose Dianne or who fell into the ‘Other’ category will not be part of this analysis of the results.

Table 1: Numerical Results

Participants	Jane	Dianne	Other
PESTs (52)	23	26	3
PSSTs (78)	32	40	6
Total (130)	55 (42%)	66 (51%)	9 (7%)

Instead, our analysis of the results will focus on the 55 participants (42%) that chose Jane and her explanation. More specifically, 23 of the 52 PESTs (44%) and 32 of the 78 (41%) of the PSSTs chose Jane and her explanation, which, as was the case with Dianne, shows little difference between the percentages associated with the PESTs and PSSTs. The 55 participants who chose Jane and her explanation did not have similar justifications for why Jane’s response and explanation was considered correct. As such, the 55 responses from those individuals who chose Jane have been further categorized in Table 2 below.

Table 2: Numerical Results within Jane Responses

Reference	order	no order	order & question
PESTs (23)	15	8	8
PSSTs (32)	30	2	13
Total (55)	45	10	21

Jane responses were organized into two distinct categories: those responses that referenced order and those responses that did not reference order. Of the 55 participants that chose Jane’s response, 45 (82%) referenced order in their justification and 10 (18%) did not reference order in their response justifications. More specifically, of the 23 PESTs that chose Jane’s response, 15 (65%) referenced order and 8 (35%) did not make reference to order and, of the 32 PSSTs that chose Jane’s response, 30 (94%) referenced order in the justifications and 2 (6%) did not reference order in their justifications, which represents a departure from the previous even split seen between PESTs and PSSTs.

Refining “Jane” responses a step further, of the 45 people that referenced order in their response justifications, 21 individuals (47%) also made reference to the question, that is, what is the probability that a three-child family has two daughters and one son, in their response justifications. More specifically, 8 of the 15 PESTs (53%) and 13 of the 30 PSSTs (43%) specifically referenced order and the question. The 21 responses referencing both order and the question, which represent, concurrently, 16% (21/130) of all participants, 38% (21/55) of those who chose Jane’s response and 47% (21/45) of those who chose Jane’s response and referenced order in their justifications, are featured in the analysis of results.

Analysis of Results

Given the consistency associated with all 21 of the responses that referenced both order and the question, 5 of the 8 responses from the PESTs and 5 of the 13 responses from the PSSTs, that is, 10 in total, are presented for analysis.

PESTs Response Justifications

In what follows, we analyse five exemplary responses, from Sam, Rebecca, Carla, Ernie and Woody, which all evidence an appeal to ignorance. We begin by considering the responses of Carla and Woody.

Carla: Dianne's explanation gives possibilities of 2 girls and one boy plus birth order possibilities – this is not what the question asked.

Woody: Dianne reuses some of the possibilities multiple times. GGB is the same as GBG. The question is not asking anything about the order in which they were born.

As italicized in the responses from Carla and Woody above, both individuals reference that birth order should not be taken into consideration. On the one hand, Woody, declaring that “GGB is the same as GBG,” is implicitly declaring that order does not matter. Carla, on the other hand, is more explicit in declaring that Dianne's explanation calculates the possibilities “plus” the birth order, which can be interpreted to mean that the order is in addition to what the question is asking. Both Woody and Carla, however, are quite clear in declaring why they have concluded that birth order does not matter. Essentially, both individuals make it clear that the question does not “ask” about the order. As seen in the responses from Sam, Rebecca and Ernie, which are presented below and similarly italicized as above, they, too, reference that the question does not “say anything” or “mention” or “never asked” (respectively) about birth order.

Sam: This explanation is better because the question doesn't say anything about birth order. It just wants to know the probability of 2 daughters and 1 son; whether or not this occurs as DDS, DSD, or SDD does not matter. I think they have a 1/4 chance of the 2 daughters and 1 son outcome.

Rebecca: There are no repetitions of the number of each sex of child in Jane's. The question doesn't mention birth order and therefore there is no need to consider that GGB and BGG is the same thing in answering this particular question.

Ernie: The question never asked about order so the only total is assumed and therefore needed ~ disregard order patterns so 4 possible outcomes.

Further, the responses from Sam, Rebecca and Ernie indicate that there is no need to consider birth order or one can, as stated by Ernie, “disregard the order patterns.” Alternatively stated, for all three responses, the question does not make reference to order mattering and, as such, order does not matter, which, ultimately, leads to choosing Dianne and her explanation as correct.

Considered from within an appeal to ignorance framework, the responses from Sam, Rebecca and Ernie and, further, from Carla and Woody (and the other 3 PESTs who referenced both order and the question in their responses), all note that the question does not provide evidence that order matters (i.e., there is no evidence for p) and, as a result, order does not matter (i.e., therefore $\text{not-}p$), which, ultimately, predicates a justification for why Dianne's response and her explanation are correct. Similar results are found within the responses from the PSSTs.

PSSTs Response Justifications

In what follows, we analyse five exemplary responses, from Frasier, Eddie, Robin, Paul and Glen, which also all evidence an appeal to ignorance. We first consider the responses of Paul and Eddie.

Paul: There are four different combinations that are possible because it didn't specifically say that order mattered. So generally, you can have 2 sons, 1 daughter; 2 daughters, 1 son; 3 sons; 3 daughters in any order.

Eddie: There is no specification as to what order the daughters & sons have to be born in. Therefore, there is only four possible outcomes causing a one in four chance.

As seen in the responses from Paul and Eddie, they make reference to the question not specifying that order mattered. Worthy of note, we are inferring in these particular responses that “it” for Paul and “there is no specifications” for Eddie are implicit references to the question. Working from this inference, for both Paul and Eddie, the reason that there are only four possible outcomes or that the different events can happen in any order are predicated on the question not providing evidence that order mattered. The responses from Frasier, Robin and Glen, presented below, are more explicit in their reference to the question.

Frasier: The question does not state the order of the siblings matters → they simply want a 2 girl + 1 boy family. Dianne has multiples of the same outcome such as DDS, and DSD.

Robin: The question did not ask what the probability is that the family has 2 daughters and one son in that order, so there are only 4 possible outcomes.

Glen: The question does not specify that the order of the children matters, they just want 2 daughters and a son. It shouldn't matter what order they come out in. As far as the question is concerned, the DDS, DSD, SDD are all the same outcome.

In fact, the above three responses are quite explicit in declaring that the questions does not “state”, “ask” or “specify” that the order of the children matters. Further, and working from the notion that the question does not specify that the order matters, all three participants conclude that the order does not matter, albeit in different ways (e.g., “multiples of the same outcome”; “there are only four possible outcomes”; “DDS, DSD, SDD are all the same outcome”). Alternatively stated, for the responses of Frasier, Robin and Glen, the question does not specify that order of the children matters and, as such, order does not matter, which, ultimately, leads to choosing Dianne and her explanation as correct.

Considered from within an appeal to ignorance framework, the responses from Paul, Eddie, Frasier, Robin and Glen (and the other 8 PSSTs who referenced both order and the question in their responses) all note that the question does not provide evidence that order matters (i.e., there is no evidence for p) and, as a result, order does not matter (i.e., therefore *not-p*), which, ultimately, acts as a justification for why Dianne's response and her explanation are correct.

Concluding Remarks

As demonstrated in the analysis of results, all 10 responses that were analyzed can be framed within the informal logical fallacy known as an appeal to ignorance. More specifically, all 10 responses made reference, whether implicit or explicit, to the question not “stating”, “asking”, “indicating” or “declaring” that order mattered (i.e., there is no evidence for p) and, as such, the responses further concluded that the order (of the outcomes) does not matter (i.e., therefore *not-p*), which was represented differently by different individuals and which led, ultimately, to their decision to choose Dianne's response and explanation. Although only 10 responses were presented in the analysis of results, we further note, based upon the striking similarities between the 10 responses presented and the 11 responses not presented, that the informal logical fallacy, known as an appeal to ignorance, accounts for 100% (21/21) of the participants whose responses referenced both order and the question, which also represents, concurrently, 47% (21/45) of the responses who chose Jane's response and referenced order in their justifications, 38% (21/55) of those responses who chose Jane's response and 16% (21/130) of all the participants involved in the current research.

Discussion

Research into the teaching and learning of probability and probabilistic thinking has focused on accounting for normatively incorrect, sometimes inexplicable responses to a variety of probabilistic tasks. Stemming from these investigations, a number of theories, models and frameworks have been developed to account for and to make sense of particular responses. Traditionally, this particular domain of research has been focused heuristic and informal reasoning. More recently, an emerging thread of research has (re)opened informal logical fallacies as a fresh perspective to account for certain response justifications. While it has been established that the fallacy of composition is able to account for incorrect responses to comparisons of relative likelihood (Chernoff & Russell, 2011a, 2011b, in press), it had not been determined to what extent informal logical fallacies can describe response justifications to other probabilistic tasks other than relative likelihood comparisons and which other fallacies could be utilized. Building upon this emerging thread of research, this article has demonstrated that another informal logical fallacy, other than the fallacy of composition, is able to account for particular responses to probabilistic tasks. More specifically, in this case, an appeal to ignorance can be added to the fallacy of composition as another particular informal logical fallacy that is able to account for certain responses to probabilistic

tasks. In other words, it can be argued that this article further strengthens the use of logical fallacies as a new area of investigation for future research on probabilistic thinking, the teaching and learning of probability and teachers' probabilistic knowledge. Despite what can be considered as early "success" with the use of particular informal logical fallacies, for example, the fallacy of composition and an appeal to ignorance, more research will determine to what extent logical fallacies play a part of teachers' and students' knowledge of probability. Speaking inductively for a moment, it appears that we are off to a good start.

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